# Matching to Produce Information: A Model of Self-Organized Research Teams*

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August 22, 2021

**Abstract**

In recent decades, research organizations have brought the "market inside the firm" by allowing workers to sort themselves into teams. How do research teams form absent a central authority? We introduce a model of team formation in which workers first match and then non-cooperatively produce correlated signals about an unknown state. Our analysis identifies matching inefficiencies arising from two channels. First, productive teams composed of workers producing complementary information may form at the expense of excluded workers who must form relatively unproductive teams consisting of workers producing substitutable information. Second, even when productive teams are efficient, they need not form; a worker in such a team may prefer to join a less productive team if she can exert less effort in this deviating team. We discuss the implications of these results for organizational design.

**Keywords:** Matching, Teams, Information Acquisition, Correlation

**JEL Classification:** C78, L23, D83

*We thank Aislinn Bohren, SangMok Lee, George Mailath, Steven Matthews, and Andy Postlewaite for support throughout the project. We also thank participants at the UPenn Micro Theory Lunch, UPenn Micro Theory Seminar, the 28th Jerusalem School in Economic Theory, the 2017 Summer School of the Econometric Society, the 2018 ICGT at Stony Brook, and the 2018 LAMES Conference. In particular, we have benefited from the comments of Yeon-Koo Che, In-Koo Cho, Jan Eeckhout, Ben Golub, Daniel Hauser, Annie Liang, Eric Maskin, Stephen Morris, Rakesh Vohra, and Leaat Yariv. Declarations of interest: none.

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# 1 Introduction

## 1.1 Background

Self-organized teams are playing an increasingly important role in economic activity. From 1987 to 1996, the fraction of Fortune 1000 firms with workers in self-managed work teams rose from 27 percent to 78 percent (Lawler, Mohrman and Benson (2001) and Lazear and Shaw (2007)). More recently, a 2016 survey of more than 7,000 executives in over 130 countries indicates that organizations are increasingly operating as a network of teams in which workers engage in self-directed research (Deloitte, 2016). These human resources trends are particularly important in organizations such as Universities (Wuchty, Jones and Uzzi (2007)) and large technology companies, like Google and Amazon, that rely on flexible internal labor markets in order to take advantage of informational complementarities among workers with diverse backgrounds. Yet while the free-ridership problem within teams has garnered considerable theoretical attention (see, for instance, Holmström (1982), Legros and Matthews (1993), and Winter (2004)), less has been devoted to the study of how moral hazard within teams affects matching. Furthermore, to our knowledge, no existing work studies this interaction in the context of the production of information.

The case of the Danish hearing-aid manufacturer Oticon illustrates well these broad trends in research and development, as well as the incentive problems that arise when decision making is delegated to productive actors themselves (see Foss (2003) for a comprehensive account). In 1987, Oticon lost almost half of its equity when its competitors began selling cosmetically superior devices. In an attempt to regain its competitive advantage, Oticon re-structured its research department, replacing vertical, hierarchical production with horizontal, project-based team production (Foss (2003) coins this organizational form a *spaghetti organization*). Beyond cosmetic changes to the office spaces — desks were no longer permanent and were located in large open spaces — there was extensive delegation of decision rights. Most notably, employees chose which projects (teams) they would join and project managers had discretion over employee compensation.

At first, these organizational changes were profitable. Eliminating hierarchies and allowing workers to lead their own teams enabled the firm to take advantage of the existing information dispersed among its workers (Kao, 1996).[1] However, new problems

[1]Oticon's CEO commented that decentralization "improved markedly [Oticon's] ability to invent new

arose. First, some teams were far better than others "in terms of how well the team members worked together and what the outcome of team effort was" (Larsen, 2002). Second, competition meant that "anybody [at a project] could leave at will, if noticing a superior opportunity in the internal job market" (Foss, 2003). These problems eventually led Oticon to introduce a company-wide employee stock option program and selectively intervene in the assignment of roles to workers within teams, designating particular workers as project managers.

While a prominent example, Oticon is not the only company to have experimented with decentralized research teams and had problems. In 2012, the multibillion-dollar video-game developer Valve publicly released a New Employer Handbook describing the company's non-hierarchical organizational structure. Valve's co-founder adopted this approach in the hope of spurring the company's research and innovation (Keighley, 2020). But, once again, decentralization led to new problems. First, talented workers refused to leave prestigious projects, and it became hard for other projects to recruit them. Second, the flat management model gave workers latitude to "minimize their work" because of the lack of "checks and balances" (Grey, 2013). In 2014, GitHub introduced a middle-management level to supervise its previously unsupervised allocation system of workers to teams (Rusli, 2014). More recently, in 2016, Medium abandoned its use of holocracy, a system "designed to move companies away from rigid corporate structures and toward decentralized management and dynamic composition" (Doyle, 2016).

## 1.2 This Paper

We posit a model of moral hazard and matching in the context of information production to better understand the managerial problems faced by firms that decentralize information production and to rationalize management solutions observed within companies like Oticon. In the setting we study, workers form teams (match) in order to forecast the value of a Gaussian state. Each worker then acquires any number of costly Gaussian signals about it. After observing all signals produced within a team, each team guesses the state and each worker receives a payoff proportional to the quadratic distance between her team's forecast and the state realization.

The matching environment features imperfectly transferable utility (Legros and Newman (2007)); one worker cannot compensate another for producing more or less signals

ideas, concepts, and make use of what [Oticon] actually [had]" (Kao, 1996). In particular, the firm was able to revive old projects that later turned out to be profitable.

than her. The literature on matching with nontransferabilities (see, for instance, Farrell and Scotchmer (1988)) has argued that social convention and social norms might rule out unequal division of surplus even if transfers within teams are permitted. Indeed, we need not look further than our own profession to see that economics scholars receive equal credit for joint work, even if work is not divided equally. More to the point, our interest is in matching inefficiency inside of firms, which is precluded once transfers are permitted; analyzing the nontransferability setting illuminates the nature of transfers and/or institutional changes required to restore efficiency.

We identify two major channels leading to matching inefficiency. First, productive teams composed of workers producing complementary information may form at the expense of excluded workers who must form relatively unproductive teams composed of workers producing substitutable information. Second, productive teams composed of workers producing complementary information may not form even when efficient; a worker in such a team may prefer to join a less productive team if, in this deviating team, she can exert sufficiently less effort. We then rationalize compensation practices at Oticon as responses to such inefficiencies.

### 1.3 Overview of Analysis

The formal analysis proceeds as follows. First, we characterize the (Pareto-Efficient Nash) equilibrium correspondence of the signal-acquisition game played within teams. Our characterization consists of cutoff values on the (state-conditional) pairwise correlation between workers' signals. Intuitively, more positively correlated signals contain more redundant information. Thus, the marginal value of producing a signal when one's teammate has already produced one is decreasing in correlation. It follows that, if the cost of producing a signal is small enough, there is a cutoff above which there is a unique asymmetric equilibrium, and another cutoff below which there is a unique symmetric equilibrium. More subtly, when signals are not too revealing, there is a third, intermediate cutoff above which all equilibria are asymmetric and below which there is at least one symmetric equilibrium (Proposition 1).

We then study the welfare efficiency of self-enforcing matchings and effort levels, i.e. core allocations. For a fixed strategy profile in which each worker produces at least one signal, minimizing pairwise correlation maximizes team productivity. Hence, one might guess that forming teams composed of workers with the lowest feasible pairwise correlations is efficient. But this need not be the case; matching such workers might cause excluded workers to form highly unproductive teams composed of workers with high pairwise correlations. As workers in highly productive teams have an incentive to match, any self-enforcing matching is inefficient. We call this phenomena *Stratification Inefficiency*, which coheres with the common observation inside of flat organizations that there is a large amount of inequality in the productivity of research teams.

Sometimes, however, a team composed of workers with a low pairwise correlation need *not* form even when it is efficient. A worker in such a team may prefer to match with another worker with whom she has a *higher* pairwise correlation if in that team she can produce relatively fewer signals than her partner in equilibrium. Moral hazard thus generates an additional sorting inefficiency, which we call *Asymmetric Effort Inefficiency*. Hence, while Stratification Inefficient core allocations feature too much inequality in productivity *across* teams, Asymmetric Effort Inefficient core allocations feature too much inequality of effort *within* teams, supporting accounts indicating an unequal amount of work between employees in the same team.

To highlight the theoretical relevance of these inefficiencies, we show that each occurs in an open set of correlation parameters for any convex cost function satisfying an Inada-type condition (Proposition 2). Our formal definitions and proofs reveal two important insights relevant to our motivating applications. First, whenever a core allocation is Stratification Inefficient, there is no other efficient core allocation (Observation 1). Hence, in the absence of transfers, Stratification Inefficiency is a robust phenomenon that can only be eliminated by actively assigning workers to teams. We suggest alternative solutions to this inefficiency when transfers are allowed and matching is decentralized. One such solution— providing employees with joint incentives— matches the organizational response of Oticon, who instituted an employee stock option program. Second, in many cases, when there is an Asymmetric Effort Inefficient core allocation, there is multiplicity and an efficient core allocation exists as well. That an efficient core allocation exists suggests a simple resolution to incentive problems: make particular workers more responsible for team output (Observation 2). Then, opportunities to free ride can be eliminated so that the efficient outcome can be obtained as a core allocation. We relate this observation to management practice at Oticon.

### 1.4 Related Literature

*Team Theory*. Chade and Eeckhout (2018) study the optimal assignment of workers to teams in the same (canonical) Gaussian environment that we consider, but with two important differences: (i) each worker produces exactly one signal within a team and (ii) utility is transferable. In our environment, in contrast to (i), workers can acquire any number of signals and, in contrast to (ii), utility is non-transferable. This first difference means that the utility of workers in a team are affected not only by their pairwise correlation, but also the number of signals each worker (endogenously) acquires (see Lemma 1, which subsumes the formula for the value of a team in Chade and Eeckhout (2018) when each worker acquires a single signal). The second difference allows us to study the impact of moral hazard on sorting, a "relevant open problem with several economic applications" (Chade and Eeckhout, 2018). Our analysis, consequently, focuses on the *efficiency* of equilibrium teams as opposed to their assortativity, as is the focus of Chade and Eeckhout (2018).[2]

An additional difference between our setup and that of Chade and Eeckhout (2018) is that they assume that signals between workers possess a *common* correlation parameter, but differ in variance, whereas we assume the opposite. We make this assumption to capture research settings in which workers are identical in their level of "expertise", but may come from different backgrounds. Our work, therefore, contributes to the literature on diversity in teams, i.e. Prat (2002), Hong and Page (2001), and Hong and Page (2004).[3] In particular, Asymmetric Effort Inefficient core allocations are characterized by excessive homogeneity, i.e. high correlation, within teams. Our results thus illustrate a new channel through which moral hazard can cause homogenous teams to form even when they are suboptimal.

*Sorting and Bilateral Moral Hazard.* Legros and Newman (2007) consider general two-sided matching environments in which, for each matched pair, there is an exogenously specified utility possibility frontier.[4] Our paper joins a small literature that considers

[2]As the latter question is of independent interest, however, in Online Appendix A we discuss how endogenous effort might affect the equilibrium assortativity of teams. Fixing the signal structure of Chade and Eeckhout (2018), we show that, once effort choice is endogenous, optimal matching must simultaneously diversify, while incentivizing effort.

[3]Prat (2002) finds conditions under which a team should be comprised of homogenous information structures when these information structures are priced according to market forces. Hong and Page (2004) and Hong and Page (2001) consider the performance of heterogeneous non-Bayesian problem solvers. In contrast, we consider the *endogenous* formation of teams by Bayesian workers within a firm with a fixed information structure.

[4]A well-known application of this framework is to risk-sharing within households. Legros and Newman (2007) and Chiappori and Reny (2016) show that if couples share risk efficiently, then all stable matchings are negative assortative. Gierlinger and Laczó (2018) show that if the assumption of perfect risk-sharing is relaxed, then positive assortative matching can occur. Schulhofer-Wohl (2006) finds necessary and sufficient conditions for preferences under which risk-sharing problems admit a transferable utility representation.

matching settings in which the utility possibility frontier of each matched pair is affected by the presence of bilateral moral hazard.[5] Kaya and Vereshchagina (2015) study one-sided matching between partners who, after matching, play a repeated game with imperfect monitoring (due to moral hazard) and transfers. While moral hazard limits the achievable joint surplus attainable by a matched pair, transfers ensure that the Pareto-frontier is linear, i.e. payoffs are transferable. Hence, stable matchings exist and (constrained) efficiency is ensured by standard arguments, in contrast to our setting.[6]

Vereshchagina (2019) studies two-sided matching between financially-constrained entrepreneurs in the presence of bilateral moral hazard and incomplete contracts; entrepreneurs can only sign contracts under which the realized revenue is split between the partners according to an equity-sharing rule.[7] Non-transferability of output gives rise to inefficient positive sorting through the following channel: wealthy entrepreneurs, whom contribute more resources to joint production, are willing to form partnerships with poor entrepreneurs only if they receive a high equity share. But, joint surplus maximizing equity shares may be constant across all partnerships. Hence, wealthy entrepreneurs prefer to match even if the overall benefit of re-matching with poor entrepreneurs is large. The logic behind inefficiency thus resembles that of Stratification Inefficiency.[8]

Finally, Kräkel (2017) considers a very different channel through which moral hazard leads to inefficient endogenous sorting. He studies an environment in which a firm posts an initial contract that determines both wages and a sorting protocol (workers either endogenously sort into teams or are randomly assigned to teams). The firm then receives interim information about the efficiency of the matches formed and can re-negotiate the initial contract. Under endogenous sorting, workers may form inefficient teams in order to force the firm to re-negotiate the initial contract.

*Correlation and Information Acquisition.* More broadly, our analysis of the information acquisition game played within teams is related to recent work defining notions of com-

[5]Wright (2004), Serfes (2005), Serfes (2007), and Sperisen and Wiseman (2016) study the assortativity of stable matchings in the presence of one-sided moral hazard, i.e. principals matching agents.

[6]Kaya and Vereshchagina (2014) study a special case of their model in which workers form partnerships that may involve "money burning" to provide incentives. They then ask whether workers would prefer to work for an entrepreneur, i.e. hire a budget-breaker, as in Franco, Mitchell and Vereshchagina (2011) to avoid this problem. Chakraborty and Citanna (2005) consider a model similar to that of Kaya and Vereshchagina (2015) in which partners play asymmetric roles.

[7]Two-sidedness again ensures that a stable matching exists, in the sense of Legros and Newman (2007), unlike in our setting.

[8]We note, however, that there is no analog to Asymmetric Effort Inefficiency in her model. A related, earlier contribution is that of Sherstyuk (1998), who shows that equal-sharing equity rules may preclude efficient heterogeneous partnerships.

plementary and substitutable information. In the environment we consider, lower correlation implies higher complementarity in terms of the value of information. Börgers, Hernando-Veciana and Krähmer (2013) define signals as complements or substitutes in terms of their value across *all* decision problems, therefore requiring stronger conditions. Liang and Mu (2020) adapt the definition of Börgers, Hernando-Veciana and Krähmer (2013) to a multivariate Gaussian environment and use it characterize the learning outcomes of a sequence of myopic players.

# 2 Model

## 2.1 Environment

There are four workers, indexed by the set $\mathcal{N} := \{1,2,3,4\}$, who form teams of two. Each team completes a project. This project involves guessing a state $\theta$, which has a Gaussian distribution with mean $\mu_\theta$ and variance $\sigma_\theta^2$. Each worker in a team can obtain unbiased, conditionally independent Gaussian signals with variance $\sigma^2$. However, the signals of workers in the same team are correlated; $\rho_{ij} \in [-1,1]$ is the state-conditional correlation coefficient between worker $i$'s and worker $j$'s signal when in the same team.

The final assignment of workers to teams is described by a matching function $\mu : \mathcal{N} \to \mathcal{N}$ such that the teammate of worker $i$'s teammate, $j$, is $i$ —that is, if $j = \mu(i)$, then $\mu(j) = i$. Let $\mathcal{M}$ denote the set of all such functions. After teams have been formed, each worker $i$ simultaneously and independently chooses a number of signals to produce, $n_i \in \mathbb{N} \cup \{0\}$, at cost $c(n_i)$, where $c : \mathbb{N} \cup \{0\} \to \mathbb{R}$ is an increasing function satisfying increasing marginal costs, i.e. $c(n) - c(n-1) \geq c(n-1) - c(n-2)$ for any $n \geq 2$. To rule out uninteresting cases, we also assume the Inada-type conditions:

$$c(0) = 0 \quad \text{and} \quad c(1) < \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma^2} \min\{\sigma_\theta^2, \sigma^2\}.$$

These conditions ensure that at least one worker has an incentive to produce at least one signal and that, if $\rho = -1$, both workers have incentive to produce a single signal and perfectly learn the state.

The correlation structure at the signal-acquisition stage captures the economics of a situation in which joint and simultaneous effort is affected by complementarities, while unilateral effort is not. In particular, we interpret $n_i$ as a decision by worker $i$ to produce a single signal in each of $n_i$ "periods", starting from period 1; if $n_i \geq n_j > 0$, then workers

$i$ and $j$ produce signals jointly in periods $t \in \{1, ..., n_j\}$. Hence, signals drawn in these periods are conditionally correlated according to $\rho_{ij}$. If $n_i > n_j$, however, then worker $i$ produces a signal alone in periods $t \in \{n_j + 1, ..., n_i\}$. Consequently, in these periods, workers cannot exploit correlation between signals to learn about the state. Figure 1 depicts the case in which $n_i = 3$ and $n_j = 2$.

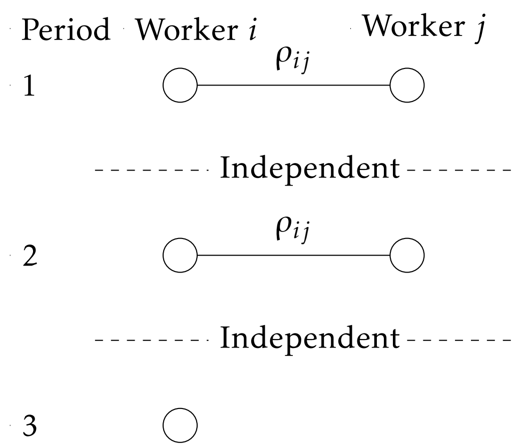



Figure 1: Signal structure when $n_i = 3$ and $n_j = 2$.

Finally, after observing the signal realizations of every team member, each team takes an action $a^* \in \mathbb{R}$ to minimize the expected value of a quadratic loss function. Formally,

$$a^* \in \arg\min_{a \in \mathbb{R}} E_\theta \left[ (a - \theta)^2 \mid x^S \right],$$

where $x^S$ denotes the concatenation of signals observed in the team.

## 2.2 Solution Concepts

A signal-acquisition strategy for worker $i$ is a function mapping teammate identity to a non-negative integer, $n_i : \mathcal{N} \to \mathbb{N} \cup \{0\}$.[9] Given a strategy for each player, we denote the profile of signals chosen in team $(i, j)$ by $n(i, j) := (n_i(j), n_j(i))$. The payoff to worker $i$ in team $(i, j)$ given the strategy profile $n(i, j)$ is

$$v_i(n(i,j); \rho_{ij}) := -E_x \left[ \min_{a \in \mathbb{R}} E_\theta \left[ (a - \theta)^2 \mid x^S \right] \right] - c(n_i(j)). \tag{1}$$

To ease notation, we denote $n_i(j)$ and $n_j(i)$ by $n_i$ and $n_j$ and drop the dependence of $v_i$ on $\rho_{ij}$ whenever there is no confusion that $j$ is $i$'s teammate.

[9]We consider pure strategies for ease of interpretation and tractability.

The strategy spaces for each player, $\mathbb{N} \cup \{0\}$, and the payoff functions defined in Equation 1 constitute a normal-form game—call it the **Production Subgame**. To account for pre-play communication, in each team $(i,j)$, we require that the strategy profile $n^*(i,j)$ is a **Pareto-Efficient Nash Equilibrium (PEN)** of the Production Subgame.

For the two-stage game, we use the core as our solution concept, the standard solution concept in the literature on matching with imperfectly transferable utility.[10] Informally, a matching function and a point in the utility possibility frontier for each matched pair is in the core if no pair can match and pick a point in their utility possibility frontier that makes both strictly better off. In our model, the utility possibility frontier of a matched pair (team) corresponds to the set of PEN payoffs in that team given their correlation coefficient. Hence, the core is defined as follows.

**Definition 1.** *A matching $\mu \in \mathcal{M}$ and a collection of PEN, $N^* = \{(n_i^*, n_j^*)\}_{i \in \mathcal{N}, j=\mu(i)}$, is in the **core** if there does not exist a matching, $\hat{\mu} \in M$, a worker $k$ with match $\ell = \hat{\mu}(k)$, and a PEN $(\hat{n}_k, \hat{n}_\ell)$ for which:*

$$v_k(\hat{n}_k, \hat{n}_\ell) > v_k(n_k^*, n_{\mu(k)}^*), \text{ and}$$

$$v_\ell(\hat{n}_k, \hat{n}_\ell) > v_\ell(n_\ell^*, n_{\mu(\ell)}^*).$$

## 2.3 Brief Discussion

We comment briefly on our modeling of the production subgame and our solution concept. See Section 5 for further discussion.

Diminishing marginal returns and complementarity are separate forces shaping workers' information acquisition strategies in our model. While acquiring more signals reduces the marginal productivity of acquiring one's own signals, correlation between signals across workers captures the degree of complementarity of information. These are plausible forces in information-producing teams that, nevertheless, are not easy to analyze separately in reduced-form production models. Our specification allows us to analyze how moral hazard is shaped by complementarities, holding fixed individual marginal returns.

In a previous version of this paper, we considered the case in which there were $N$ workers and workers were allowed to work alone. The analysis in the current paper largely generalizes to that setting. However, in that setting, existence of the core is not

[10]See Legros and Newman (2007) for a general definition in two-sided environments and Kaya and Vereshchagina (2015) for a definition in a one-sided environment.

guaranteed due to roommate-type problems.[11,12]

# 3 Production Subgame Analysis

## 3.1 Preliminaries

Because each worker's payoff function is quadratic, her optimal action given any signal realization is the posterior mean. Hence, her expected payoff when signals are costless is the negative posterior variance. Lemma 1 states these observations and provides a closed-form solution for the posterior variance. The proof is standard, but provided in Appendix A.1 for completeness.[13]

**Lemma 1.** *Suppose workers $i$ and $j$ form a team and acquire $(n_i, n_j)$ signals with $n_i \leq n_j$. Each worker's optimal action is $a = E(\theta \mid x)$ and the expected payoff of worker $i$ is*

$$v_i(n_i, n_j) = Var(\theta \mid (n_i, n_j)) - c(n_i),$$

*where $x$ is the concatenation of realized signals and*

$$Var(\theta \mid (n_i, n_j)) := \begin{cases} 0 & \text{if } i \neq j, n_i > 0, n_j > 0 \text{ and } \rho_{ij} = -1 \\ \left(\left(\frac{2n_i}{1+\rho_{ij}} + (n_j - n_i)\right)\sigma^{-2} + \sigma_\theta^{-2}\right)^{-1} & \text{otherwise.} \end{cases}$$

The pairwise correlation coefficient $\rho := \rho_{ij}$, for $i \neq j$, measures the substitutability of information between workers: as $\rho$ increases, the value of working together decreases. For intuition, consider the extreme cases. When $\rho = -1$, by producing $(1,1)$ signals, a team can match the state by choosing an action equal to the sample average. On the other hand, when $\rho = 1$, working together to produce $(1,1)$ signals is equivalent to having only one worker produce a signal.

[11] The core is not *generally* guaranteed to exist in our setting, but exists in all of our proof constructions. We use the core as our solution concept for expositional simplicity.

[12] To overcome the existence problem, we defined a new solution concept— Coalitional Subgame Perfect Equilibrium — in which PEN are fixed in all teams as a part of the solution concept. This limits the set of payoffs achievable by a deviating pair of workers and enabled us to prove existence. As every core allocation is also a Coalitional Subgame Perfect Equilibrium, our inefficiency analysis applies under this alternative solution concept as well. We are currently developing this solution concept further for general one-sided matching environments.

[13] Proposition 1 of Chade and Eeckhout (2018) considers the posterior variance associated with a team composed of $n$ workers each acquiring a single signal. All signals have the same pairwise correlation. Our formulation captures differences in correlation across "periods" of information acquisition. This, in turn, affects strategic behavior, as outlined in Section 3.2.

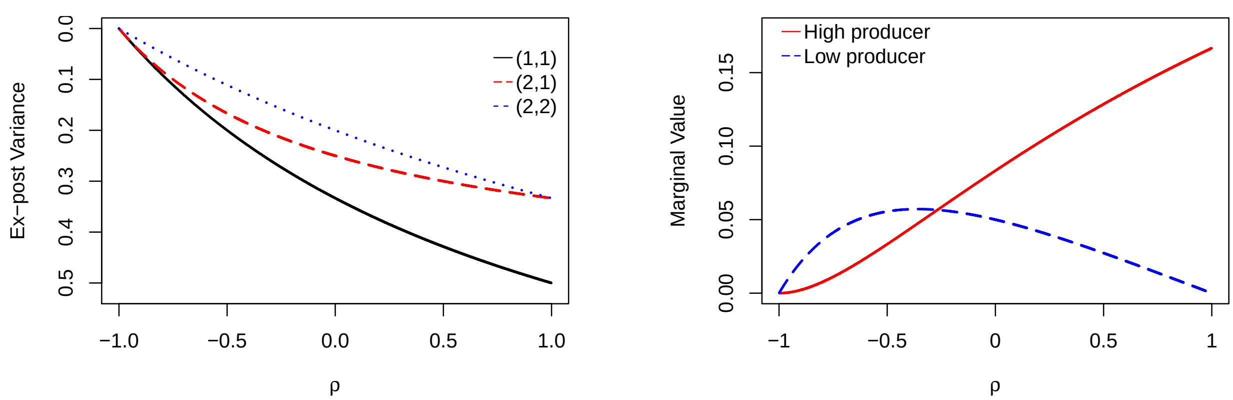
## 3.2 The Marginal Value of Information

To characterize PEN, we define and analyze the **marginal value of information** to worker $i$ of producing a signal in the $n_i$-th period given that worker $j$ produces a signal in the first $n_j$ periods. This marginal benefit corresponds to the reduction of the ex-post variance generated by the last signal:

$$MV(n_i; n_j, \rho) \equiv Var(\theta \mid (n_i - 1, n_j)) - Var(\theta \mid (n_i, n_j)).$$

If $n_i \geq n_j$, we call worker $i$ a **high producer**. If the inequality is strict, we call worker $j$ a **low producer**.

Figure 2a illustrates the posterior variance $Var(\theta \mid (n_i, n_j))$ for different correlations, $\rho$, and strategy profiles, $(n_i, n_j)$, in the case in which $\sigma = \sigma_\theta = 1$. In Figure 2a, the difference between the dashed red line and the solid black line is the marginal value of information to a high producer of producing a signal in period two, while the difference between the dotted blue line and the dashed red line is the marginal value of information to a low producer of producing a signal in period two, given that the high producer is already producing one in the first two periods. The former difference is represented by the solid, red line in Figure 2b, while the latter is represented by the dashed, blue line in Figure 2b.




(a) Ex-post Variance.

(b) Marginal Value of Second Signal.

Figure 2: Ex-post Variance and Marginal Values.

We make three observations about the figures, which generalize beyond the parameterization we consider, and which we exploit in proving our main characterization result. First, the marginal value of information to the high producer is strictly increasing in $\rho$. This happens because the value of the information obtained from working together with the low producer in previous periods *decreases*. By concavity of the information production function, the marginal value of information left to learn increases.

Second, the marginal value of information to a low producer is non-monotonic in $\rho$. Indeed, we see the *difference* between the blue line and red line in Figure 2a is non-monotonic, and so the blue line in Figure 2b is hump-shaped. The marginal value of the low producer is increasing in an initial region for the same reason the high producer's marginal value is increasing; when $\rho$ increases, the value of work done together in past periods decreases and so the marginal value of information left to learn increases. However, there is another effect to consider. When $\rho$ increases, the value of working together with the high producer in a future period *decreases*– the high producer and low producer's information is less complementary. After an interior cutoff value $\tilde{\rho}$, the second effect dominates and the marginal value of information to the low producer decreases.

Third, the marginal value of a high producer is higher than the marginal value to a low producer above a negative cutoff value, $\hat{\rho}$, which holds whenever signals are sufficiently noisy, i.e. $\sigma^2 \geq \sigma_\theta^2$. This will prove important in our characterization of within team equilibrium.

### 3.3 PEN Characterization

We now prove existence of PEN and link within-team correlation to the symmetry of PEN strategies.

**Proposition 1.** *There exist cutoff values $-1 < \rho^* \leq \rho^{**} < 1$ on a team's pairwise correlation, $\rho$, such that the following properties hold:*

1. *If $\rho \leq \rho^*$, then there is a unique PEN. It is symmetric and each worker produces a strictly positive number of signals.*

2. *If $\rho > \rho^{**}$, then, generically, there is a unique PEN up to the identity of each worker. In it, one worker produces a strictly positive number of signals and the other produces none.*

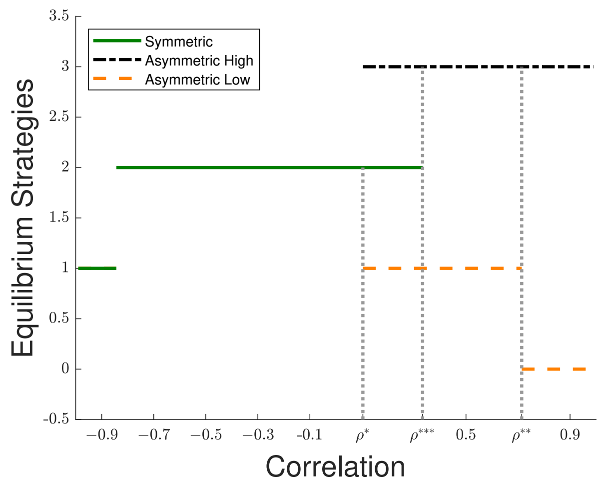
*If $\sigma^2 \geq \sigma_\theta^2$, then there exists another cutoff value $\rho^{***} \in [\rho^*, \rho^{**}]$ for which the following properties hold:*

3. *If $\rho \in (\rho^*, \rho^{***}]$, then there is at least one symmetric and one asymmetric PEN.*

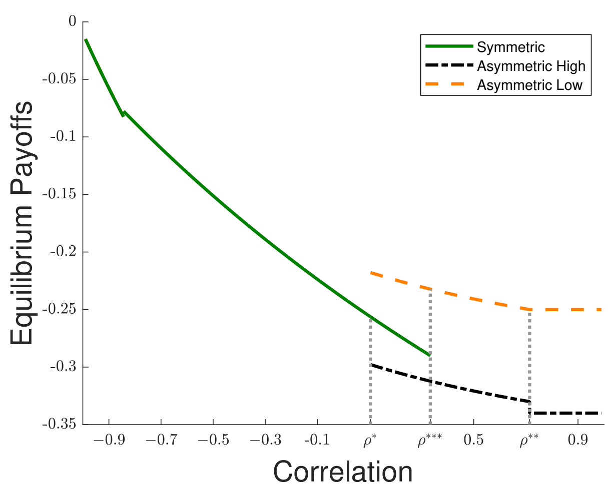
4. *If $\rho > \rho^{***}$, then all PEN are asymmetric.*

We prove existence of a Nash equilibrium by proving that the game played within a team is a potential game. Hence, it is guaranteed to have a pure strategy Nash equilibrium by Corollary 2.2 of Monderer and Shapley (1996). We then observe that the set of Nash equilibria is finite. Hence, there exists one that is Pareto undominated, i.e. a PEN exists.

With existence take care of, we study the symmetry of the PEN correspondence. Figure 3a illustrates our results in a case in which $\sigma^2 \geq \sigma_\theta^2$, so that all four properties of of Proposition 1 apply; on the $x$-axis is the team's correlation parameter, $\rho$, and on the $y$-axis are the equilibrium strategies of each worker in the team. The solid green line indicates the strategy taken by each worker when a symmetric PEN is played given $\rho$, the equally-spaced dashed orange line indicates the strategy of a low producer in an asymmetric PEN, and the asymmetrically-spaced dashed black line indicates the strategy of a high producer in an asymmetric PEN.



(a) Equilibrium Correspondence.



(b) Payoff Correspondence.

Figure 3: PEN Correspondence with $\sigma^2 = \sigma_\theta^2 = 1$ and $c(n) = 0.01n^2$.

The first property of Proposition 1 is that there exists a correlation threshold, $\rho^*$, below which there is a unique PEN and it is symmetric, with each worker producing at least one signal. The proof is simple: By the Inada-type condition on the cost of effort, when $\rho = -1$, there is a unique PEN in which both workers have a strict incentive to produce exactly one signal. As posterior variance is continuous in $\rho$, it follows that there is a neighborhood above $\rho = -1$ in which $(1,1)$ is the unique PEN. Figure 3a, however, illustrates that this need not be the only symmetric PEN is played. In the Figure, $\rho^* \approx .10$ and there is an interval below it at which $(2,2)$ is the unique PEN. We remark that $\rho^*$ can either be above or below zero, depending on the cost function.

The second property of Proposition 1 is that there exists a correlation threshold, $\rho^{**}$, above which there is a unique PEN (up to identity permutations) in which one worker produces a strictly positive number of signals and the other produces none. Again, by the Inada-type condition on the cost of effort, when $\rho = 1$, at least one worker produces at least one signal and a low producer has a strict incentive *not* to match the high producer's signal (it provides no benefit at that value). Hence, by continuity of posterior variance, there is a neighborhood around $\rho = 1$, in which, generically, the unique PEN in which only one worker produces a strictly positive number of signals.[14] In Figure 3a, $\rho^{**} \approx .71$ and above this value the unique PEN involves one worker producing three signals and the other producing none.

The final two properties of Proposition 1 indicate that, if $\sigma^2 \geq \sigma_\theta^2$, there is a third correlation cutoff $\rho^{***} \in [\rho^*, \rho^{**}]$ above which all PEN are asymmetric and below which there is at least one symmetric and one asymmetric PEN (provided that $\rho \in [\rho^*, \rho^{***}]$). In Figure 3a, $\rho^{***} \approx 0.33$; above it there are asymmetric PEN $(1,3)$ and $(0,3)$ and below it there is both a symmetric PEN $(2,2)$ and an asymmetric PEN $(1,3)$.

The intuition behind these properties is subtle. When signals are sufficiently noisy, $\sigma^2 \geq \sigma_\theta^2$, then the marginal value of information for a low producer is maximized at a correlation $\tilde{\rho}$ strictly below the value at which the marginal value of information for a high producer exceeds that of the low producer, $\hat{\rho}$, as in Figure 2b. Hence, increasing $\rho$ past $\hat{\rho}$ increases the marginal value to the high producer while decreasing the marginal value to the low producer. Behavior then coheres with intuition; higher correlations drive equilibria to be asymmetric because high producers have an increasing incentive to acquire more information, while low producers have a decreasing incentive to match the signals produced by high producers.[15]

[14] The genericity qualifier rules out the case in which a worker is indifferent between two positive integers when $\rho = 1$ and her teammate produces zero signals.

[15] If $\hat{\rho} < \tilde{\rho}$, a counterintuitive phenomena emerges. In this case, there is a region in which increasing $\rho$

# 4 Inefficient Matching

Our analysis of the Production Subgame yields two important insights. First, fixing a strategy profile within teams, reducing correlation increases the value of information the team generates. Hence, there is a tendency for workers with a low pairwise correlation to match, ignoring effort costs. Second, increasing correlation decreases the symmetry of equilibria; as signals become more substitutable, the marginal value of matching a high producer's signal decreases. Hence, the existence of nontransferable effort costs may tempt workers with a low pairwise correlation to join less productive teams.

We now show how these two within-team properties influence the efficiency of sorting of workers into teams. Our exposition will utilize the payoff correspondence in Figure 3b, which corresponds to the payoffs in the PEN discussed in the previous section. We emphasize that our matching analysis is nontrivial both because of the existence of nontransferability of utility between workers and because there does not exist a natural "ranking" of types; all six correlation parameters between workers matter for equilibrium sorting.

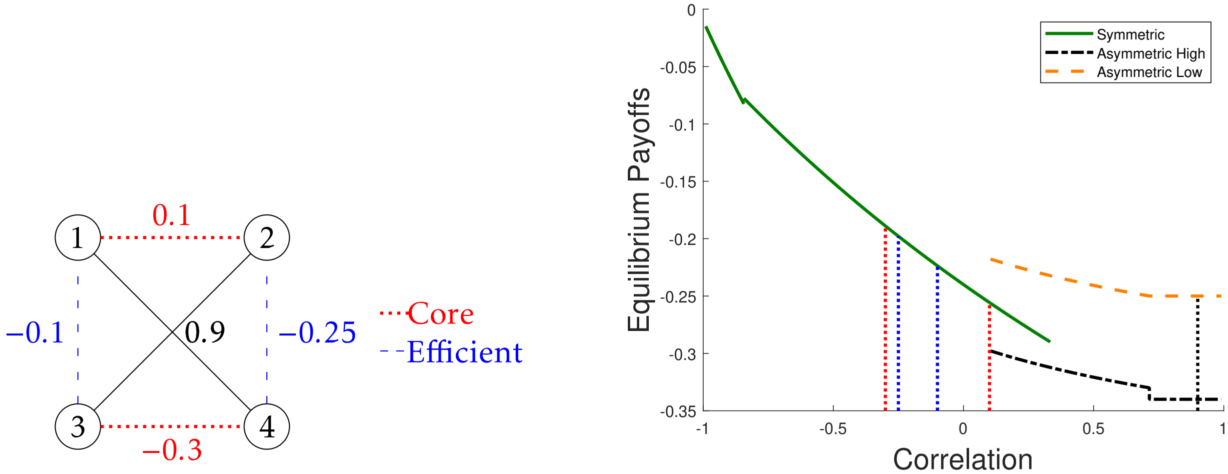
## 4.1 Stratification Inefficiency

We first exposit an inefficiency, Stratification Inefficiency, that arises because two highly productive workers, i.e. workers with low pairwise correlation, match at the expense of the two excluded workers, whom must form a less productive team with a relatively high pairwise correlation.

Suppose, for simplicity, that the parameters are as in Figure 3, so that all properties of Proposition 1 apply. Suppose further that the network in Figure 4a describes the correlation matrix; numbers next to adjacent edges depict pairwise correlation. Then, the unique PEN in teams composed of workers connected by dotted or dashed lines is the symmetric profile $(2,2)$ and the unique PEN (up to identity) in teams composed of workers connected by solid lines is the asymmetric profile $(0,3)$.[16] Corresponding payoffs are depicted in Figure 4b.

past $\hat{\rho}$ increases the marginal value for both the high producer *and* the low producer. Hence, if an asymmetric equilibrium is played at some correlation $\rho$ above $\hat{\rho}$, but below $\tilde{\rho}$, it may be the case that for a higher correlation a symmetric equilibrium may be played. Why? The increase in the value of information left to learn for the low producer might induce her to match the high producer's signal. If this happens, the high producer's incentive to produce another signal may decline enough so that she does not produce another one herself. For such an example, we direct the reader to Online Appendix C.

[16] This can be seen by referring back to Figure 3.




(a) Correlation Parameters.

(b) PEN Payoffs.

Figure 4: Stratification Inefficiency.

We argue that the unique matching in the core pairs worker 1 (worker 3) and worker 2 (worker 4), while the efficient matching pairs worker 1 (worker 2) and worker 3 (worker 4). To see why worker 3 and worker 4 must form a team in any core matching, see that worker 3 and worker 4 each obtains a higher payoff in a team together than in any PEN in any other team (see Figure 4b). Indeed, their pairwise correlation, -0.3, is the smallest among all feasible teams and all teams other than $(1,4)$ and $(2,3)$ play the same PEN. In addition, in teams $(1,4)$ and $(2,3)$, neither worker 3 nor worker 4 can obtain a higher payoff as a free-rider in an asymmetric PEN. It follows that worker 3 and worker 4 must match, so that worker 1 and worker 2 have no other choice, but to match.[17]

While we have argued that in the unique core allocation, worker 1 (worker 3) and worker 2 (worker 4) match, it remains to argue that matching worker 1 (worker 2) and worker 3 (worker 4) is welfare improving. Why might this be the case? Though the most productive team, $(3,4)$, forms in the core matching, this comes at the cost of preventing workers 1 and 2 from joining teams with significantly lower pairwise correlations. In particular, while the team $(1,2)$ produces positively correlated signals, the teams $(1,3)$ and $(2,4)$ do not. It turns out that the gain in productivity obtained from re-matching worker 1 with worker 3, and worker 2 with worker 4, outweighs the cost of disrupting the most productive team $(3,4)$.

We now formalize the logic just described and define our first notion of inefficiency.

[17]This example generalizes to the case in which worker 1 and worker 2 can work alone, provided that forming a team has a positive cost. We considered this case in an earlier version of this paper.

**Definition** 2 (Stratification Inefficiency)**.** *A core allocation* $(\mu, N^*)$ *is* ***Stratification Inefficient*** *if*

1. *there exist two workers* $i, j \in \mathcal{N}$*,* $i \neq j$*, for which which* $\mu(i) = j$ *and* $v_\ell(n^*(i,j))$ *is the highest payoff worker* $\ell \in \{i, j\}$ *can obtain in any PEN in any team; and,*

2. *there exists a matching* $\mu' \neq \mu \in \mathcal{M}$ *and a collection of PEN* $\hat{N} = \{\hat{n}(i,j)\}_{i,j=\mu(i)}$ *such that,*

$$\sum_{\ell \in \mathcal{N}} v_\ell(n^*(\ell, \mu(\ell))) < \sum_{\ell \in \mathcal{N}} v_\ell(\hat{n}(\ell, \mu'(\ell))).$$

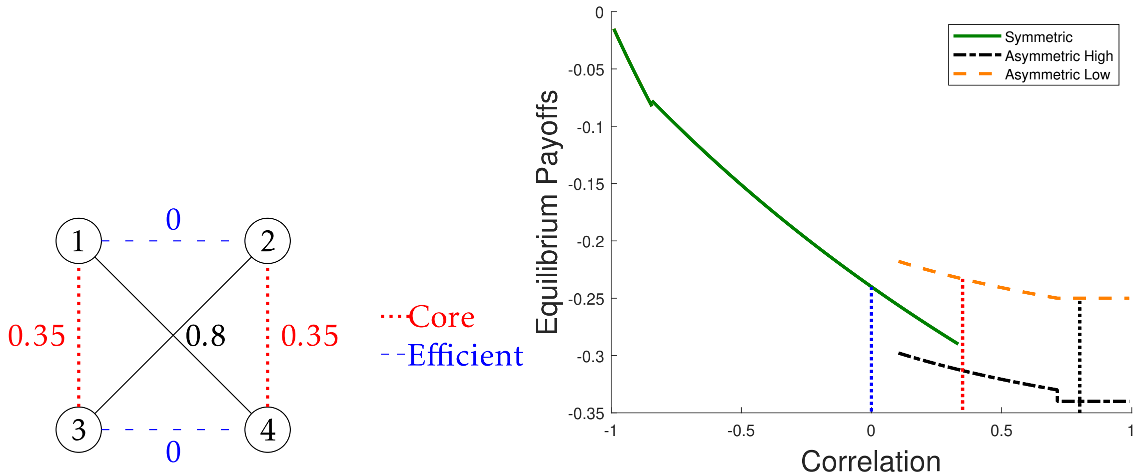
The first condition requires that, in any Stratification Inefficient matching, a pair of teammates are each as well off as in *any* other feasible team playing *any* other PEN, i.e. worker 3 and worker 4 in our example. The second condition requires that there exists another matching, i.e. $\hat{\mu}$ such that $\hat{\mu}(3) = 1$ and $\hat{\mu}(4) = 2$ in our example, and a collection of PEN in each team that increases utilitarian welfare. Stratification Inefficiency therefore arises because two (possibly highly productive) workers each obtains a higher payoff together than in any other team, but do not internalize the "externality" they generate on the productivity of other matches. An efficiency-minded manager, in contrast, prefers them not to match so that she can better exploit the entire correlation matrix.

## 4.2 Asymmetric Effort Inefficiency

Stratification Inefficiency is *not* driven by free riding. Indeed, in the inefficient matching we illustrated, each worker works as much as she would in the efficient matching. We now focus on the implications of free riding within teams for sorting across teams.

For illustration, suppose again that the parameters are as in Figure 3. But now, suppose the network in Figure 5a describes the correlation matrix. The unique PEN in teams composed of workers connected by blue dashed lines is the symmetric profile $(2,2)$, the unique PEN (up to identity) in teams composed of workers connected by dotted red lines is the asymmetric profile $(1,3)$, and the unique PEN (up to identity) in teams composed of workers connected by solid lines is the asymmetric profile $(0,3)$. Corresponding payoffs are depicted in Figure 5b.

We argue that there is a matching in the core that pairs worker 1 (worker 2) and worker 3 (worker 4), even though the efficient matching pairs worker 1 (worker 3) and worker 2 (worker 4). To see why such a matching exists, consider the incentives of worker




(a) Correlation Parameters.

(b) PEN Payoffs.

Figure 5: Asymmetric Effort Inefficiency.

1.[18] She has two relevant options: form a team with worker 2, with whom she produces uncorrelated signals, or form a team with worker 3, with whom she produces positively correlated signals.In the team with worker 2, worker 1 produces two signals in any PEN. On the other hand, in a team with worker 3, worker 1 either produces three signals (so that worker 3 produces one signal) or *one* signal (so that worker 3 produces three signals). In the case in which worker 1 produces three signals when matched with worker 3, it is clear that she would rather form a team with worker 2; not only is the value of information produced lower in the team with worker 3, she is exerting more effort. But, if worker 1 produces one signal when matched with worker 3, so that she is the low producer in that team, then she would rather form a team with worker 3; though the value of information produced is lower, she is exerting *less* effort (see Figure 5b for the payoff comparison).

So, for argument's sake, *fix* the PEN within team $(1,3)$ to be the strategy profile $(1,3)$ so that worker 1 would rather match with worker 3 than worker 2. Worker 3 would only deem such a team acceptable if she could not persuade worker 4 to form a team with her. But, fixing the PEN within team $(2,4)$ to be the asymmetric profile $(3,1)$, worker 4 prefers to work with worker 2 (worker 4 has the "same" options as worker 1). Hence, there is a core matching in which worker 1 matches worker 3 (and worker 2 matches worker 4) because worker 3 (worker 2) has no better option. The core allocation is inefficient, however, because not only does the total value of information produced in the firm increase

[18] Again, teams $(1,4)$ and $(2,3)$ can never form in any matching in the core; the two workers whom produce all the signals in each team are better off forming a deviating team.

by forming teams $(1,2)$ and $(3,4)$, but total effort costs decrease weakly.[19]

We again formalize the logic just described and define our second notion of inefficiency.

**Definition 3** (Asymmetric Effort Inefficiency). *A core allocation* $(\mu, N^*)$ *is **Asymmetric Effort Inefficient** if*

1. *there exist two workers* $i, j \in \mathcal{N}$, $i \neq j$, *for which* $\mu(i) = j$, *and a PEN* $\hat{n}(i,i')$, $i' \neq i$, *satisfying*
$$n_i^*(j)\hat{n}_{i'}(i) < \hat{n}_i(i')n_j^*(i);$$
*and,*

2. *there exists a matching* $\hat{\mu} \in \mathcal{M}$ *satisfying* $\hat{\mu}(i) = i'$ *and a collection of PEN* $\hat{N} = \{\hat{n}(i,j)\}_{i,j=\mu(i)}$, *including* $\hat{n}(i,i')$, *such that*
$$\sum_{\ell \in \mathcal{N}} v_\ell(n^*(\ell,\mu(\ell))) < \sum_{\ell \in \mathcal{N}} v_\ell(\hat{n}(\ell,\mu'(\ell))).$$

To understand the definition, consider again the example. Let $(i,j) = (1,3)$ and $(i',j') = (2,4)$. The manager prefers to match worker 1 with worker 2 because there is a symmetric PEN inside the team, $\hat{n}(1,2) = (2,2)$, in which worker 1 exerts *relatively* more effort than her partner when compared to the "on-path" PEN, $n^*(1,3) = (1,3)$. In particular, $\frac{n_1^*(3)}{n_3^*(1)} = \frac{1}{3} < 1 = \frac{\hat{n}_1(2)}{\hat{n}_2(1)}$ so that, by cross-multiplying, we see that the first inequality of the definition is satisfied. The second part of the definition ensures that, upon re-matching worker 1 and worker 2 and fixing $\hat{n}(1,2)$, the manager can select a PEN and matching of the other two workers so that utilitarian welfare increases.

## 4.3 Analytical Result

We prove now that the inefficiencies identified in the previous two sections require neither the specific cost function nor parameters we have assumed.

**Proposition 2.**

1. *For any signal variance* $\sigma^2$, *prior variance* $\sigma_\theta^2$, *and cost function* $c$ *satisfying the assumptions of the environment, there is an open set of correlation parameters for which there is a Stratification Inefficient core allocation.*

[19] Recall, the cost of effort is increasing for an individual worker and the total number of signals produced in each team in the efficient and inefficient matching is the same.

2. *If $\sigma^2 \geq \sigma_\theta^2$, then, for any cost function c satisfying the assumptions of the environment, there is an open set of correlation parameters for which there is an Asymmetric Effort Inefficient core allocation.*

The proof of Proposition 2 makes full use of our characterization of within-team equilibria to construct correlations leading to inefficiency. In particular, to construct a Stratification Inefficient core allocation, we choose four pairwise correlations strictly below $\rho^*$, the cutoff below which there is a unique and symmetric PEN, and all others above it. As long as there is no free-riding opportunity for the workers with the lowest correlation, they must match in any core allocation. However, if the other two workers have a sufficiently high correlation, then re-matching workers can improve welfare.

To construct an Asymmetric Effort Inefficient core allocation, we observe that, as long as $\sigma^2 \geq \sigma_\theta^2$, there is an open set of correlations above $\rho^*$ for which there is an asymmetric PEN of the Production Subgame. We then pick a worker, say worker 1, and two pairwise correlations— $\rho_{13}$, in this open set, and $\rho_{12}$ below $\rho^*$—so that worker 1 prefers to free ride in the team with worker 3 than to match worker 2. If $\rho_{12}$ is small enough, however, the sum of utilities in the team $(1,2)$ exceeds that in $(1,3)$, as in our illustrative example.[20]

## 4.4 Implications for Organizational Design

We now discuss how a central planner, i.e. a manager, might overcome Stratification Inefficiency and Asymmetric Effort Inefficiency using transfers and/or within-team equilibrium selection. Our goal is not to conduct an exhaustive analysis of optimal contract design, which is beyond the scope of the current paper and requires careful consideration of the legal environment in which the firm operates. Instead, we illuminate the ways in which performance evaluation and management systems must be changed in order to prevent matching inefficiency when decision rights within a firm are delegated to workers themselves.

[20]A careful reader should note that while we have demonstrated that Stratification Inefficiency and Asymmetric Inefficient core allocations occur in a range of non-trivial scenarios, we have *not* argued that they are the only sources of inefficiency in our model. This claim is *false* precisely because welfare inefficiency may exist *within* a team. In particular, a PEN may be selected within a team that is welfare dominated by another PEN. As the focus of our analysis is on inefficient *sorting*, however, we do not attempt to characterize such inefficiencies. We conjecture, but have not proven that, taking care of within-team inefficiency, the inefficiencies we have identified are exhaustive.

### 4.4.1 Overcoming Stratification Inefficiency

In the absence of transfers, Stratification Inefficiency is a phenomenon that can only be eliminated by actively intervening in the assignment of workers to teams. In particular, if two workers obtain a higher payoff together than in any other team playing any PEN, there is no way to select PEN within teams to induce either to form an efficient team.

**Observation 1.** *If a Stratification Inefficient core allocation exists, then equilibrium selection within teams cannot restore efficiency of sorting into teams.*

If utility is transferable within teams, then, as is well-known, the efficient allocation of workers to teams can be decentralized as a core allocation. Here is one efficient and budget-balanced transfer scheme: Charge one of the two workers obtaining the highest payoff in any PEN in any team an amount making her indifferent between her desired teammate and her efficient teammate, and similarly for this efficient teammate. For example, returning to Figure 4, consider a Vickrey-Clarke-Groves type mechanism in which worker 3 pays worker 1 his externality on him, $v_1(2,2;\rho_{13}) - v_1(2,2;\rho_{12})$, if he matches with worker 4 (an inefficient teammate). This simple transfer is enough to ensure efficiency; worker 1 and worker 3 have an incentive to match, forcing worker 2 and worker 4 to match.

Alternatively, a manager might construct a transfer scheme involving *joint performance evaluation*, i.e. payments to each worker that depend on total output in the firm. For instance, returning to the example presented in Figure 4, suppose that worker 3 receives a bonus based on the productivity of the team they are *not* a part of, $(i,j)$. In particular, suppose she receives a bonus of

$$b = \alpha \cdot (a_{(i,j)} - \theta_{(i,j)})^2 > 0,$$

where $a_{(i,j)}$ is team $(i,j)$'s action, $\theta_{(i,j)}$ is the realization of the state team $(i,j)$ is guessing, and

$$\alpha > \frac{v_2(2,2;\rho_{24}) - v_2(2,2;\rho_{12})}{v_3(2,2;\rho_{34}) - v_3(2,2;\rho_{13})} \iff$$

$$\underbrace{v_3(2,2;\rho_{34}) + \alpha \cdot v_2(2,2;\rho_{12})}_{\text{Payoff for worker 3 when matching with worker 4}} < \underbrace{v_3(2,2;\rho_{13}) + \alpha \cdot v_2(2,2;\rho_{24})}_{\text{Payoff for worker 3 when matching with worker 1}}.$$

Under this scheme, incentives to produce information within teams are identical to the setup studied in our previous analysis (the group component of compensation is a constant in worker 3's utility function). However, by construction, the additional transfer worker 3 receives based on the productivity of the team they are *not* a part of encourages them to match efficiently.

In the case of Oticon, joint performance evaluation schemes, in the form of stock options, appeared to be particularly important in alleviating Stratification Inefficiency (whose existence is suggested by the quotation of Larsen (2002) in the Introduction). In Foss (2003)'s account of the company's history, the author notes that Oticon's CEO, Lars Kolind, introduced an employee stock program, in which

> "shop floor employees were invited to invest up to 6.000 Dkr (roughly 800 USD) and managers could invest up to 50.000 Dkr (roughly 7.500 USD). Although these investments may seem relatively small, in Kolind's view they were sufficiently large to significantly matter for the financial affairs of individual employees; therefore, they would have beneficial incentive effects. More than half of the employees made these investments."

Our analysis rationalizes these compensation schemes as a way of resolving Stratification Inefficient matching.[21]

#### 4.4.2 Overcoming Asymmetric Effort Inefficiency

In contrast to Stratification Inefficiency, Asymmetric Effort Inefficiency may possibly be prevented by equilibrium selection within teams and in the absence of transfers. For example, returning to Figure 5, if a manager designates worker 1 as a high producer, then she can enforce an efficient core allocation. In particular, if we choose a PEN in team $(1,3)$ so that worker 1 is the team high producer, rather than worker 3, then worker 1 would rather form a team with worker 2. As worker 2 prefers this arrangement to the case in which she matches with worker 4 and is the high producer, worker 1 and worker 2 match, leaving worker 3 and worker 4 to match. Additionally, in the open set of parameters we identify in our proof of Proposition 2, a manager can always improve welfare by forcing worker 1 to be the high producer when matched with worker 3, i.e. selecting a PEN in which she exerts relatively more effort than worker 3. Then, the efficient matching is itself in the core.

**Observation 2.** *If an Asymmetric Effort Inefficient core allocation exists, then an efficient core allocation may also exist. Provided that it exists, a manager can enforce the efficient core allocation by selecting which PEN is played within each team.*

[21] An interesting extension of our analysis would take into account workers' strategic decisions to invest in stock options prior to the matching stage.

We remark that equilibrium selection powers are plausible in many organizational contexts; while it may be costly to assign all workers to teams, it may not be so costly to manage the behavior of particular workers. For instance, a manager might assign specific responsibilities to individual workers. Foss (2003)'s account of Oticon indicates exactly this type of involvement. In particular, in 1996, Oticon developed a "Compentence Center" which appointed and oversaw project leaders.

# 5 Discussion

Our paper is a first step towards understanding how research teams form absent a central authority and in the absence of transfers. We shed light on how workers' incentives for effort within teams are affected by their informational complementaries and therefore impact equilibrium sorting. Our analysis uncovers two plausible forces leading to inefficient sorting. First, workers producing complementary information may match and force excluded workers to form highly unproductive teams composed of workers producing substitutable information. Hence, there is too much inequality in productivity *across* teams. Second, even when it is efficient for a team composed of workers producing complementary information to form, such a team may not arise in equilibrium if one of its members has an opportunity to form a less productive team in which she exerts relatively less effort. Hence, there is too much inequality in effort *within* teams. We discuss how a manager might alleviate each inefficiency, either using transfers or equilibrium selection within teams.

We conclude by commenting on the structure of our model leading to our results and on the extensions we have considered.

*Gaussian Environment.* We model information acquisition using a canonical quadratic-Gaussian set-up; workers obtain normally distributed signals to minimize a quadratic loss function and have normally distributed prior beliefs.[22] In this environment, the expected value of the posterior distribution simplifies to the negative posterior variance (Lemma 1). Hence, we can derive comparative statics using a closed-form utility function. Our analysis uncovers how correlation between teammates affects the symmetry of the PEN correspondence–that we can order equilibria by symmetry in terms of correlation is the crucial property for our main inefficiency results. In Online Appendix D, we consider a

[22] Our results generalize to the case in which the joint distribution of signals and states for any number of draws is elliptical with finite second moments. In this case, the conditional expectation is still linear in signals and our characterization results will possess the same qualitative features.

binary signal, binary state environment, as is common in applied theoretical work. While the intuition that perfect negative correlation leads to perfect learning does not hold (because such correlation is statistically infeasible), it is still the case that low correlation in state-conditional signals is desirable. Consequently, the marginal value of a draw, and hence equilibrium predictions, satisfy the same properties as in the Gaussian case.

*Draw Procedure.* The procedure through which workers acquire and share information possesses two features which deserve comment. First, workers choose numbers of signals simultaneously. Methodologically, we abstract from dynamic considerations in order to isolate the key property relevant for team formation–namely, the relationship between correlation and the symmetry of equilibrium strategies. Nonetheless, it is worthwhile to explore the extent to which the intuitions we have provided hold in a more complex dynamic game. Towards an answer, in Online Appendix E, we study a finite extensive form game with sequential decisions. Our main conclusion is that for many, but not all, correlations there is a Subgame Perfect Equilibrium of the sequential game that coincides with the most symmetric equilibrium of the simultaneous game. Nonetheless, it may be the case that an equilibrium of the simultaneous game is more asymmetric than the most symmetric equilibrium of the extensive form game. Hence, sequential decisions do *not* eliminate asymmetric equilibria, the driving force behind Asymmetric Effort Inefficiency.

Second, the correlation between signals differs across "periods"; if $(n_i, n_j)$ draws are taken in team $(i, j)$ with correlation $\rho$, and $n_j > n_i > 0$, then the first $n_i$ signals drawn by each worker are correlated according to $\rho$ and the last $n_j - n_i$ signals are conditionally independent. We assume that pairwise correlation affects the value of effort within, but not across, periods in order to capture the economics of a situation in which joint and simultaneous effort is affected by complementarities, while unilateral effort is not. In particular, it would *not* be equivalent to analyze a continuous choice model in which workers first choose precisions and then share a single signal. In this set-up, within and across period effects cannot be disentangled.[23]

*Size-Two Teams.* We follow the matching literature in assuming that workers form

[23] Nonetheless, a kind of "continuous draw" set-up may be imagined as follows. Suppose, relative to a single signal of fixed precision, a worker can draw many signals with lower precisions, but with the cost of information held constant. In Online Appendix F, we analyze the limit model obtained when such precisions become arbitrarily small and costs are adjusted. Workers can therefore be interpreted as choosing a real number of signals. In this limit model, the equilibrium correspondence can be ordered by symmetry using pairwise correlations as in our main characterization result, and hence our main inefficiency Propositions generalize. We do not use such a model in the main text, however, as we have not proven that the equilibrium correspondence of the sequence of discrete-draw models converges to that of the continuous-draw model.

teams with at most two workers. This restriction allows us to obtain a clean characterization of within-team equilibria; a single pairwise correlation coefficient captures intuitively the effects of skill complementarity. Extending the analysis to teams of more than two workers is not without its challenges. In subsequent work, Segura-Rodriguez (2019) shows that a team of three workers can perfectly learn the state even if each worker produces a single signal and all three signals are *highly* correlated. Characterizing within-team equilibria with many workers and exploring the implications of these equilibria for team formation is an interesting open problem we leave for future research.

*Profit-Maximizing Sorting.* Our current framework illustrates the ways in which decentralized matching within firms may be inefficient. In subsequent work, Kambhampati and Segura-Rodriguez (2020) study the problem of assigning workers to teams and designing incentive contracts in the presence of both moral hazard and adverse selection and when the manager desires to maximize profits. They characterize when creating incentives in a centralized organization becomes so costly that a profit-maximizing manager prefers to allow workers to sort themselves into teams and compensate them equally on the basis of team output alone. Nonetheless, the environment in Kambhampati and Segura-Rodriguez (2020) is simpler than the one considered in this paper and so their results do not directly apply. A complete analysis of the profit-maximizing tradeoff between centralization and decentralization in informational settings thus awaits future research.

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# A Proofs

## A.1 Proof of Lemma 1

For any measurable function $g : X \to \mathbb{R}$, where $X$ is the set of possible realizations of signals,

$$-\mathbb{E}_{x,\theta}\left[(g(x)-\theta)^2\right] \leq -\mathbb{E}_x\left[(\mathbb{E}(\theta \mid x)-\theta)^2\right] = -\mathbb{E}_x\left[\mathbb{E}_\theta\left[(\mathbb{E}(\theta \mid x)-\theta)^2 \mid x\right]\right] = -Var(\theta \mid x).$$

The inequality follows because $\mathbb{E}\left[(b-\theta)^2|x\right]$ is minimized by setting $b = \mathbb{E}[\theta|x]$. The first equality follows from the Law of Iterated Expectations. The second equality follows from the definition of conditional variance.

Let $\Sigma$ be the correlation matrix of joint signals $x$, and $1_N$ be a $N$-column vector of 1s. The likelihood function of the signals is, $p(x|\theta) = \det(2\pi\sigma^{-2}\Sigma)^{-\frac{1}{2}} \exp\Big( - \frac{1}{2}\Big[(\theta \cdot 1_N - x)'\sigma^{-2}\Sigma^{-1}(\theta \cdot 1_N - x)\Big]\Big)$ and the prior density is, $p(\theta) = (2\pi\sigma^{-2})^{-\frac{1}{2}} \exp\Big( - \frac{1}{2}\Big[(\theta - \mu_\theta)^2\sigma_\theta^{-2}\Big]\Big)$.

By Bayes rule, the posterior distribution of $\theta|x$ is proportional to,

$$\begin{aligned} p(x|\theta)p(\theta) &\propto \exp\Big( - \frac{1}{2}\Big[(\theta - \mu_\theta)^2\sigma_\theta^{-2} + (\theta \cdot 1_N - x)'\sigma^{-2}\Sigma^{-1}(\theta \cdot 1_N - x)\Big]\Big) \\ &\propto \exp\Big( - \frac{1}{2}\Big[\theta^2(\sigma_\theta^{-2} + \sigma^{-2}1_N'\Sigma^{-1}1_N) - \theta(2\mu_\theta\sigma_\theta^{-2} + \sigma^{-2}(x'\Sigma^{-1}1_N + 1_N'\Sigma^{-1}x)\Big]\Big) \\ &\propto \exp\Big( - \frac{1}{2}\Big[\theta - A\Big]'B\Big[\theta - A\Big]\Big), \end{aligned}$$

where $B = (\sigma_\theta^{-2} + \sigma^{-2}1_N'\Sigma^{-1}1_N)$, $A = B^{-1}(\mu_\theta\sigma_\theta^{-2} + \sigma^{-2}1_N'\Sigma^{-1}x)$, and the proportionality operator eliminates positive constants. Since the derived expression is the kernel of a normal distribution, $Var(\theta \mid x) = B^{-1}$.

We construct $B^{-1}$ when workers take $n_j \geq n_i$ draws. The prior covariance matrix, $\Sigma^{-1}$, is block diagonal with $n_i$ blocks of the form,

$$\Sigma_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

and $n_j - n_i$ scalar blocks each equal to 1. The inverse of a block diagonal matrix is equal to the block diagonal matrix formed by inverting each block. Then, $1_N'\Sigma^{-1}1_N$ is equal to $n_i 1_2'\Sigma_0^{-1}1_2 + (n_j - n_i)$. Since,

$$\Sigma_0^{-1} = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix},$$

we have, $1_2'\Sigma_0^{-1}1_2 = \frac{2}{1+\rho}$. Hence,

$$Var(\theta \mid n_i, n_j) = B^{-1} = \Big(\sigma^{-2}1_N'\Sigma^{-1}1_N + \sigma_\theta^{-2}\Big)^{-1} = \left(\sigma^{-2}\left(\frac{2n_i}{1+\rho} + (n_j - n_i)\right) + \sigma_\theta^{-2}\right)^{-1}.$$

Finally, if $\rho = -1$, the average of two signals equals the realized state $\theta$, and so the posterior variance is zero.

## A.2 Proof of Proposition 1

### A.2.1 Existence of Nash Equilibrium

Since information production exhibits diminishing marginal returns, eventually, the marginal value of producing a signal must be less than the marginal cost *regardless* of the behavior of one's partner. Therefore, it is without loss to bound the action space.

**Lemma 2.** *There is a positive integer $\bar{N}$ such that for each positive integer $n \geq \bar{N}$, $n$ is a not best response by worker $i$ to any strategy by worker $j$.*

*Proof.* For $n_i \leq n_j$,

$$Var(\theta \mid (n_i - 1, n_j)) - Var(\theta \mid (n_i, n_j)) = \frac{\left(\frac{1-\rho}{1+\rho}\right)\sigma^{-2}}{\left(\left(n_i \frac{1-\rho}{1+\rho} + n_j + 1 - \frac{2}{1+\rho}\right)\sigma^{-2} + \sigma_\theta^{-2}\right)\left(\left(n_i \frac{1-\rho}{1+\rho} + n_j\right)\sigma^{-2} + \sigma_\theta^{-2}\right)}$$

is strictly decreasing in $n_j$ and in $n_i$ because $\frac{1-\rho}{1+\rho} > 0$. For $n_i \geq n_j + 1$,

$$Var(\theta \mid (n_i - 1, n_j)) - Var(\theta \mid (n_i, n_j)) = \frac{\sigma^{-2}}{\left(\left(n_j \frac{1-\rho}{1+\rho} + n_i - 1\right)\sigma^{-2} + \sigma_\theta^{-2}\right)\left(\left(n_j \frac{1-\rho}{1+\rho} + n_i\right)\sigma^{-2} + \sigma_\theta^{-2}\right)}$$

is strictly decreasing in $n_i$ and in $n_j$, again because $\frac{1-\rho}{1+\rho} > 0$.

Therefore, the marginal value of worker $i$ is strictly decreasing in $n_j$, so that worker $i$'s best response is decreasing in $n_j$. We only need to prove that worker $i$'s best response to 0 draws by worker $j$ is finite. It suffices to show that there is an $n_i \in \mathbb{Z}_+$ such that $Var(\theta \mid (n_i - 1, 0)) - Var(\theta \mid (n_i, 0))$ is smaller than $c(1)$. We have

$$Var(\theta \mid (n_i - 1, 0)) - Var(\theta \mid (n_i, 0)) = \frac{1}{(n_i - 1)\sigma^{-2} + \sigma_\theta^{-2}} - \frac{1}{n_i \sigma^{-2} + \sigma_\theta^{-2}} < \frac{\sigma^2}{n_i(n_i - 1)}.$$

Then, it is sufficient to have $n_i > \frac{\sigma^2}{c(1) n_i} + 1$. When $n_i > \frac{\sigma^2}{c(1)} + 1$ we obtain the desired inequality. Define $\bar{N} \in \mathbb{N}$ as the smallest value that satisfies the inequality. □

Since we can bound the action space, we may redefine the game as a finite exact potential game to show that there exists a pure strategy Nash equilibrium.

**Lemma 3.** *There exists a pure strategy Nash equilibrium of the Production Subgame.*

*Proof.* Given that no worker optimally produces a number of signals larger than $\bar{N}$, we can redefine the Production Subgame as the finite normal form game $(\{0, 1, \ldots, \bar{N}\}^2, \{v_i, v_j\})$. Define the potential function,

$$\Phi(n_i, n_j, \rho_{ij}) = -Var(\theta \mid (n_i, n_j)) - c(n_i) - c(n_j),$$

where $\rho_{ij}$ is the correlation for team $(i,j)$. It is a potential function since

$$\begin{aligned} v_i(n_i,n_j) - v_i(n_i',n_j) &= -Var(\theta \mid (n_i,n_j)) - c(n_i) + Var(\theta \mid (n_i',n_j)) + c(n_i') \\ &= \Phi(n_i,n_j,\rho_{ij}) - \Phi(n_i',n_j,\rho_{ij}) \\ v_j(n_i,n_j) - v_j(n_i,n_j') &= -Var(\theta \mid (n_i,n_j)) - c(n_j) + Var(\theta \mid (n_i,n_j')) + c(n_j') \\ &= \Phi(n_i,n_j,\rho_{ij}) - \Phi(n_i,n_j',\rho_{ij}). \end{aligned}$$

Hence, the redefined game is a finite exact potential game and is guaranteed to have a pure strategy Nash equilibrium by Corollary 2.2 of Monderer and Shapley (1996). □

### A.2.2 Existence of Pareto-Efficient Nash Equilibrium

By Lemma 2, we can conclude that the set of Nash Equilibria is finite. Consider the subset of equilibria that maximizes worker $i$'s payoff. Choose any equilibrium that (weakly) maximizes worker $j$'s payoff within this subset. The chosen equilibrium must be Pareto-Efficient. Hence, a Pareto-Efficient Nash Equilibrium exists.

### A.2.3 Comparative Statistics Preliminary Lemmas

Lemma 4 states that the marginal value of a signal by a high producer is increasing in $\rho$.[24]

**Lemma 4** (High Producer Comparative Statics in $\rho$)**.** *For $n_i > n_j$, $MV(n_i;n_j,\rho)$ is increasing in $\rho$.*

*Proof.* For $n_i > n_j$,

$$\frac{\partial MV(n_i;n_j,\rho)}{\partial \rho} \propto \left(\left(n_j\frac{1-\rho}{1+\rho}+n_i\right)\sigma^{-2}+\sigma_\theta^{-2}\right)+\left(\left(n_j\frac{1-\rho}{1+\rho}+n_i-1\right)\sigma^{-2}+\sigma_\theta^{-2}\right)>0.$$

□

The same property does not hold for a low producer.[25] We prove the low producer's marginal benefit is strictly concave in the pairwise correlation $\rho$ and has a unique maximizer. For the following lemmas it is useful to define the signal-to-prior variance ratio $\gamma := \frac{\sigma^2}{\sigma_\theta^2}$.

[24] Recall, a *high producer* is a teammate taking weakly more draws than her partner.

[25] Recall, a *low producer* is a teammate taking strictly fewer draws than her partner.

**Lemma 5** (Low Producer Comparative Statics in $\rho$)**.** *For $n_i < n_j$ with $n_j \geq 1$, $MV(n_i+1;n_j,\rho)$ is strictly concave in $\rho$ with unique maximizer,*

$$\tilde{\rho}(n_i+1,n_j,\gamma) = \frac{\left(n_j+\gamma-\sqrt{n_i(n_i+1)}\right)^2}{-(n_j+\gamma)^2+n_i(n_i+1)}.$$

*Proof.* If worker $j$ produces $n_j$ signals, the marginal benefit of the $n+1$-th signal for worker $i$, with $n < n_j$, is equal to,

$$MV(n+1;n_j,\rho) = \frac{\frac{1-\rho}{1+\rho}}{\left(n\frac{2}{1+\rho}+n_j-n+\gamma\right)\left((n+1)\frac{2}{1+\rho}+n_j-n-1+\gamma\right)}.$$

Differentiating with respect to $\rho$,

$$\frac{\partial MV(n+1;n_j,\rho)}{\partial\rho} = \frac{2\sigma^2\left(-(n_j+\gamma)^2(1+\rho)^2+n(n-1)(1-\rho)^2\right)}{\left(2n+(n_j-n+\gamma)(1+\rho)\right)^2\left(2(n+1)+(n_j-n-1+\gamma)(1+\rho)\right)^2}.$$

Differentiating again with respect to $\rho$,

$$\begin{aligned}\frac{\partial^2 MV(n+1;n_j,\rho)}{\partial\rho^2} &\propto 4n(n+1)\left(-(n_j+\gamma)^2(1+\rho)-n(n-1)(1-\rho)\right)\\ &+n(n+1)\left[2n(n_j-n-1+\gamma)+2(n+1)(n_j-n+\gamma)+(n_j-n-1+\gamma)(n_j-n+\gamma)\right](2\rho-2)<0.\end{aligned}$$

Hence, the marginal value $MV(n+1;n_j,\rho)$ is strictly concave in $\rho$. The unique maximizer $\tilde{\rho}$ must satisfy,

$$(n_j+\gamma)^2(1+\tilde{\rho})^2 = n(n+1)(1-\tilde{\rho})^2,$$

a quadratic equation in $\rho$ with roots,

$$\rho = \frac{\left(n_j+\gamma\pm\sqrt{n(n+1)}\right)^2}{-(n_j+\gamma)^2+n(n+1)}.$$

Both solutions are negative because the denominator is negative. However, the smaller root (corresponding to the "plus" in the numerator) is less than $-1$ and therefore infeasible. Since $n+1 \leq n_j$, the other root (corresponding to the "minus" in the numerator) is greater than $-1$. Set $\tilde{\rho}(n+1,n_j,\gamma) = \frac{\left(n_j+\gamma-\sqrt{n(n+1)}\right)^2}{-(n_j+\gamma)^2+n(n+1)}$. □

We now make stepwise comparisons between the marginal value of a signal by a high producer and the marginal value of a signal by a low producer. Workers initially produce $n-1$ signals. The high producer's marginal value is the payoff of producing an $n$-th signal. The low producer's marginal value is the payoff of producing an $n$-th signal, given that the high producer already produced $n$ signals. Lemma 6 states that for any number $n \geq 1$

and signal-to-prior variance ratio $\gamma = \frac{\sigma^2}{\sigma_\theta^2}$, there is a unique correlation, $\hat{\rho}(n,\gamma)$, below which the marginal value of the high producer is less than the marginal value of the low producer, and above which the opposite holds.

**Lemma 6** (High Producer-Low Producer MV Comparison 1)**.** *Fix $n_j > n_i$ with $n_j \geq 1$ and $\gamma$. Then,*

$$\underbrace{MV(n_j; n_i, \rho)}_{\textit{Marginal Value High Producer}} < \underbrace{MV(n_i+1; n_j, \rho)}_{\textit{Marginal Value Low Producer}}$$

*if and only if,*

$$\rho < \hat{\rho}(n_i+1, n_j, \gamma) = \frac{-(\gamma + n_i + n_j) + \sqrt{(\gamma + n_i + n_j)^2 - 4(\gamma + n_j - n_i - 1)}}{2(\gamma + n_j - n_i - 1)} < 0.$$

*Proof.*

$$\begin{aligned}
&MV(n_i+1; n_j, \rho) \geq MV(n_j; n_i, \rho) \\
\Leftrightarrow &\frac{\frac{1-\rho}{1+\rho}}{\left(n_i \frac{2}{1+\rho} + n_j - n_i + \gamma\right)\left((n_i+1)\frac{2}{1+\rho} + n_j - n_i - 1 + \gamma\right)} \geq \frac{1}{\left(n_i \frac{2}{1+\rho} + n_j - n_i - 1 + \gamma\right)\left(n_i \frac{2}{1+\rho} + n_j - n_i + \gamma\right)} \\
\Leftrightarrow &\frac{1-\rho}{1+\rho}\left(n_i \frac{2}{1+\rho} + n_j - n_i - 1 + \gamma\right) \geq \left((n_i+1)\frac{2}{1+\rho} + n_j - n_i - 1 + \gamma\right) \\
\Leftrightarrow &\ 0 \geq (\gamma + n_j - n_i - 1)\rho^2 + (\gamma + n_i + n_j)\rho + 1.
\end{aligned}$$

The last inequality involves a quadratic concave function in $\rho$. The roots are:

$$\rho^+(n_i, n_j) = \frac{-(\gamma + n_i + n_j) + \sqrt{(\gamma + n_i + n_j)^2 - 4(\gamma + n_j - n_i - 1)}}{2(\gamma + n_j - n_i - 1)}$$

$$\rho^-(n_i, n_j) = \frac{-(\gamma + n_i + n_j) - \sqrt{(\gamma + n_i + n_j)^2 - 4(\gamma + n_j - n_i - 1)}}{2(\gamma + n_j - n_i - 1)}.$$

When $n_i \geq 1$ the expression inside the root is greater than $(\gamma + n_j - n_i)^2$, so that $\rho^-(n_i, n_j) < -1$ for all $n_i \geq 1$. If $n_i = 0$, the expression inside the root is equal to $(\gamma + n_j - 2)^2$. If $\gamma + n_j < 2$ then $\rho^-(0, n_j) < -1$, and if $\gamma + n_j \geq 2$, then $\rho^-(0, n_j) = -1$. Therefore, $\rho^-$ is an infeasible solution.

It is clear that $\rho^+(n_i, n_j) < 0$ for all $n_i$ and $n_j$, and

$$\rho^+(0, n_j) = \frac{-(\gamma+1) + \sqrt{(\gamma-1)^2}}{2\gamma} = \begin{cases} \frac{-2}{2(\gamma+n_j-1)} > -1 & \text{if } \gamma + n_j \geq 2 \\ \frac{2-2(\gamma+n_j)}{2(\gamma+n_j-1)} = -1 & \text{if } \gamma + n_j < 2. \end{cases}$$

Further, when $n_i \geq 1$ the expression inside the root is larger than $(\gamma + n_j - 2)^2$. Therefore, $\rho^+(n_i, n_j) > -1$ for all $n_j > n_i \geq 0$. Then $\rho^+$ is a feasible solution and we set $\hat{\rho}(n_i + 1, n_j) = \rho^+$. □

Lemma 7 states that if $\gamma$ is sufficiently large, the pairwise correlation at which the marginal value of a low producer is maximized, $\tilde{\rho}(n_i, n_i, \gamma)$, is less than $\hat{\rho}(n_i, \gamma)$. We use this property in the next section to order equilibria in terms of their symmetry.

**Lemma 7** (High Producer-Low Producer MV Comparison 2)**.** *Fix* $n_j > n_i \geq 1$*. Then for* $\gamma \geq 1$

$$\tilde{\rho}(n_i + 1, n_j, \gamma) \leq \hat{\rho}(n_i + 1, n_j, \gamma).$$[26]

*Proof.* Define the function $g(n_i + 1, n_j, \gamma) := \tilde{\rho}(n_i + 1, n_j, \gamma) - \hat{\rho}(n_i + 1, n_j, \gamma)$. We want to show that $g(n_i + 1, n_j, \gamma) \leq 0$ for any $n_j > n_i \geq 1$ and any $\gamma \geq 1$. It suffices to show that $g(n_i + 1, n_j, 1) \leq 0$ for any $n_j > n_i \geq 1$ and then show that $\frac{\partial g(n_i+1, n_j, \gamma)}{\partial \gamma} < 0$ for any $n_j > n_i \geq 1$.

We first show that $g(n_i + 1, n_j, 1) \leq 0$. Notice,

$$g(n_i+1, n_j, 1) = \frac{\left(n_j + 1 - \sqrt{n_i(n_i+1)}\right)^2}{-(n_j+1)^2 + n_i(n_i+1)} - \frac{-(1+n_i+n_j) + \sqrt{(1+n_i+n_j)^2 - 4(n_j - n_i)}}{2(n_j - n_i)} \leq 0,$$

if and only if,

$$\frac{2(n_j - n_i)\left(n_j + 1 - \sqrt{n_i(n_i+1)}\right)^2 + \left((1+n_i+n_j) - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right)\left(-(n_j+1)^2 + n_i(n_i+1)\right)}{2(n_j - n_i)\left(-(n_j+1)^2 + n_i(n_i+1)\right)} \leq 0.$$

For any $n_j > n_i \geq 1$, the denominator is negative. Hence, the expression holds if and only if,

$$2(n_j - n_i)\left(n_j + 1 - \sqrt{n_i(n_i+1)}\right)^2 + \left((1+n_i+n_j) - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right)\left(-(n_j+1)^2 + n_i(n_i+1)\right) \geq 0.$$

[26] For $n_i = 0$, $\tilde{\rho}(n_i, n_j, \gamma) = -1$, so the inequality is satisfied for any $\gamma$.

Dividing by $n_j + 1 - \sqrt{n_i(n_i+1)} > 0$, we see that the inequality holds if and only if

$$2(n_j - n_i)(n_j + 1 - \sqrt{n_i(n_i+1)}) - \left(1 + n_i + n_j - \sqrt{(1+n_i+n_j)^2 - 4(n_j - n_i)}\right)(n_j + 1 + \sqrt{n_i(n_i+1)}) \geq 0$$
$$\Leftrightarrow (n_j+1)\left(n_j - 3n_i - 1 + \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right) \geq \sqrt{n_i(n_i+1)}\left(3n_j - n_i + 1 - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right).$$

Since $n_j \geq n_i + 1$, we have that $n_j + 1 \geq 2n_i - \sqrt{n_i(n_i+1)} + 2$. So, it is sufficient to show that

$$(2n_i - \sqrt{n_i(n_i+1)} + 2)\left(n_j - 3n_i - 1 + \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right)$$
$$\geq \sqrt{n_i(n_i+1)}\left(3n_j - n_i + 1 - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right)$$
$$\Leftrightarrow (n_i+1)\left(n_j - 3n_i - 1 + \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right) - 2\sqrt{n_i(n_i+1)}(n_j - n_i) \geq 0.$$

To complete the argument, we show that (i) the left-hand side of the last expression is positive when $n_j = n_i + 1$ and (ii) increasing in $n_j$. To show $(i)$, notice that when $n_j = n_i + 1$ the expression on the left-hand side is positive if and only if

$$-n_i + \sqrt{n_i^2 + 2n_i} \geq \sqrt{\frac{n_i}{n_i+1}}$$
$$\Leftrightarrow 2(n_i+1) - \frac{1}{n_i+1} \geq 2\sqrt{n_i^2 + 2n_i}$$
$$\Leftrightarrow \frac{1}{(n_i+1)^2} \geq 0,$$

where the second and third lines are the result of taking squares on both sides and simplifying. Clearly, the last inequality always holds. To show (ii), notice that the derivative of the left-hand side with respect to $n_j$ is positive if and only if

$$(n_i+1)\left(1 + \frac{n_i + n_j - 1}{\sqrt{(n_i+n_j+1)^2 - 4(n_j - n_i)}}\right) \geq 2\sqrt{n_i(n_i+1)}.$$

Since $n_j > n_i$, the left-hand side of this expression is greater than $\frac{2(n_i+1)(n_i+n_j)}{n_i+n_j+1}$. Hence, it is sufficient to show that,

$$\frac{n_i + n_j}{n_i + n_j + 1} \geq \sqrt{\frac{n_i}{n_i+1}}$$
$$\Leftrightarrow n_j^2 \geq n_i(1 + n_i),$$

where the second line is the result of taking squares on both sides and simplifying. But, this holds is as long as $n_j \geq n_i + 1$. We have thus completed the proof that $g(n_i+1, n_j, 1) \leq 0$.

To show that $\frac{\partial g(n_i+1,n_j,\gamma)}{\partial\gamma} < 0$, we first observe that for all $\gamma \geq 1$,

$$\begin{aligned}\frac{\partial\hat{\rho}(n_i+1,n_j,\gamma)}{\partial\gamma} &\propto \left(-1+\left((\gamma+n_j+n_ji)^2-4(\gamma+n_j-n_i-1)\right)^{-0.5}2(\gamma+n_i+n_j-2)\right)2(\gamma+n_j-n_i-1)\\ &\quad -2\left(-(\gamma+n_j+n_i)+\sqrt{(\gamma+n_j+n_i)^2-4(\gamma+n_j-n_i-1)}\right)\\ &= 2(2n_j+1)\sqrt{(\gamma+n_j+n_i)^2-4(\gamma+n_j-n_i-1)}+2(n_i+n_j+\gamma)^2-8(n_j+1)>0.\end{aligned}$$

The inequality follows because the first term is positive and the second term minus the third term is non-negative: $2(n_i+\gamma)^2-8$ if $n_i=0$, and greater than $2(2n_i+1)^2-8(n_i+1)$ if $n_i>0$. Second, we observe that for all $\gamma \geq 1$,

$$\frac{\partial\tilde{\rho}(n_i+1,n_j,\gamma)}{\partial\gamma} \propto n_i(n_i+1)-(n_j+\gamma)\sqrt{n_i(n_i+1)}<0,$$

since $\sqrt{n_i(n_i+1)} > n_i$, and $n_j \geq n_i+1$. □

### A.2.4 Proof of Proposition 1 part 1. and 2.

Since $c(1) < \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma^2}\min\{\sigma_\theta^2,\sigma^2\}$, for $\rho=-1$ and correlations close to it, both workers produce at least one signal. Since $MV(2;1,\rho)$ is close to 0, for correlations close to $-1$, no worker has an incentive to produce a second signal when both are producing a single signal. Hence, there is a threshold $\rho^* > -1$ below which the unique equilibrium is symmetric.

As $\rho$ approaches 1, the marginal benefit of matching the first signal of one's teammate, $MV(1;1,\rho)$, approaches zero. By continuity and monotonicity of $MV(1;1,\rho)$ in $\rho$, there exists a unique $\rho^{**} < 1$ such that $MV(1;1,\rho^{**}) = c(1)$. Since, by the proof of Lemma 2, $MV(1;n,\rho) = c(1)$ is decreasing in $\rho$ when $n \geq 1$, the low producer in the team has no incentive to match the high producer's first signal. Hence, for $\rho > \rho^{**}$ one worker produces zero signals and the other produces a strictly positive number of signals. This PEN is unique up to identity, except in the case in which the high producer is indifferent between two numbers of signals.

### A.2.5 Proof of Proposition 1 part 3. and 4.

For this proof, we use the *Sequential Response Algorithm*:

1. Set $(n_i^0, n_j^0) = (0,0)$ and $t=1$.

2. If $MV(t;n_j^{t-1},\rho) > c(t)-c(t-1)$, set $n_i^t = n_i^{t-1}+1$ and move to step 3, replacing $t$ with $t+1$. If not, set $n_i^t = n_i^{t-1}$ and move to step 4, replacing $t$ with $t+1$.

3. Set $n_j^t = \arg\max_{n \leq n_i^t} Var(\theta \mid (t,n)) - c(n)$ and go back to step 2, replacing $t$ with $t+1$.

4. (Complement Effect) Set $(n_i^t, n_j^t) = (n_i^{t-1}+1, n_j^{t-1}+1)$ if (i) both workers are made weakly better off and (ii) the resulting profile is a Nash equilibrium. If either (i) or (ii) is not satisfied, set $(n_i^t, n_j^t) = (n_i^{t-1}, n_j^{t-1})$ and move to step 5, replacing $t$ with $t+1$. Else, repeat step 4, replacing $t$ with $t+1$.

5. (Substitution Effect) Consider the profile $(n_1^{t-1}+1, n_2^{t-1}-n)$, where $n_2^{t-1}-n$ is a best-response by worker $j$ given $n_1^{t-1}+1$ subject to the constraint that $0 \leq n \leq n_2^{t-1}$. Set $(n_1^t, n_2^t) = (n_1^{t-1}+1, n_2^{t-1}-n)$ if (i) both workers are made weakly better off and (ii) the resulting profile is a Nash equilibrium. Then, repeat step 5, replacing $t$ with $t+1$. If either (i) or (ii) are not satisfied, exit the algorithm and return $(n_1^{t-1}, n_2^{t-1})$.

The algorithm terminates in finite time for the following two reasons. First, the algorithm eventually exits the loop between step 2 and step 3 because the marginal value of a signal approaches zero and costs are increasing. Second, the algorithm eventually exits step 4 and step 5 because, by Lemma 2, there is a positive integer above which worker 1 no longer wants to produce a signal, no matter the number of signals produced by worker 2.

**Lemma 8.** *The Sequential Response Algorithm finds a PEN that minimizes* $|n_i - n_j|$.

*Proof.* We first claim that if step 4 is reached in iteration $t+1$, then $(n_i^t, n_j^t)$ is a Nash equilibrium. To see this, note that, after step 1, the algorithm cycles between step 2 and step 3. We make two observations about this cycle. First, worker $j$ either exits the loop having never produced a signal or she matches worker $i$'s signal the first time step 3 is reached.[27] Second, if worker $j$ does not match worker $i$'s signal in step 3, then she never increases the number of signals she acquires in any future iteration in which step 3 is reached (her marginal value decreases each time step 3 is reached).

When step 4 is reached, worker $j$ does not have a profitable deviation downwards by construction. Checking that worker $i$ has no profitable downward deviation is more involved. First, suppose step 3 was never reached. Then, worker $i$ exits having produced zero signals and cannot reduce the number of signals she produces further. Second, suppose step 3 was reached at least once. If during last time step 3 was reached worker $j$

[27] To understand why, consider the first time step 2 is reached. Worker $i$ either (i) produces zero signals or (ii) produces one signal. In case (i), the algorithm proceeds to step 4 with both workers having produced zero signals. In case (ii), the algorithm proceeds to step 3. If worker $j$ then matches worker $i$'s first signal, we are done. Otherwise, worker $j$ best responds by producing zero signals. But if she produces zero signals, she must exit the loop having produced zero signals; each future iteration at which step 3 is reached, the marginal value of producing a strictly positive number of signals decreases.

matched worker $i$'s signal, then symmetry ensures that worker $i$ has no profitable deviation. If during the last time step 3 was reached, worker $j$ best responded by weakly decreasing the number of signals she produced, then the marginal value of information for worker $i$ is larger after $j$'s decision than before. Once again, worker $i$ has no profitable downward deviation.

We now check for profitable upward deviations. When step 4 is reached, worker $i$ does not have a profitable deviation upwards by construction. Moreover, $n_j^t \leq n_i^t$. If $n_j^t = n_i^t$, then $j$'s incentives are the same as $i$'s and so she has no profitable deviation upwards. If $n_j^t < n_i^t$, however, she has no incentive to produce $n_j^t + 1$ signals by construction. Moreover, since the marginal value of information is decreasing in the number of signals she produces, she does not want to produce any larger number of signals either.

then our second observation about the algorithm suggests that $n_j^t + 1$ was not profitable. But, then, any larger number of signals is also not profitable. Hence, worker $j$ has no profitable deviation upwards. We have thus established that the profile entering step 4 is a Nash equilibrium.

Step 4 and step 5 ensure that the algorithm finds a PEN. That this PEN is the most symmetric follows from the incremental construction in the cycle between step 2 and step 3.

□

The proof consists of two steps. First, we argue that if $(n_1, n_2)$ with $n_1 > n_2$ is a PEN for correlation $\rho$, then for correlation $\rho' > \rho$ there exists an equilibrium $(n_1', n_2')$ with $n_1' \geq n_1$ and $n_2' \leq n_2$. Second, we argue that if there is no symmetric PEN at $\rho$, then for $\rho' > \rho$ there is no symmetric PEN as well. These two properties together imply the result.

First, suppose that for correlation $\rho$ there is a PEN $(n_1, n_2)$ with $n_1 > n_2$. Then, it has to be that

$$MV(n_1; n_2, \rho) \geq MV(n_2 + 1; n_1, \rho).$$

Lemma 6 and Lemma 7 imply that $\rho > \hat{\rho}(n_2 + 1, n_1, \gamma) > \tilde{\rho}(n_2 + 1, n_1, \gamma)$. Hence, by Lemma 5, for any correlation $\rho' > \rho$, $MV(n_2+1; n_1, \rho') < MV(n_2+1; n_1, \rho)$. Pick $\rho' > \rho$ and start the *Sequential Response Algorithm* at step 3. Since the marginal value of draw $n_2 + 1$ is smaller at $\rho'$ than at $\rho$, the optimal response of worker 2 to $n_1$ draws is smaller than or equal to $n_2$ when the correlation is $\rho'$ instead of $\rho$. Continuing with the algorithm, we find a PEN $(n_1', n_2')$. Since the number of signals produced by worker 1 can only increase throughout the algorithm and the marginal value of a signal by worker 2 decreases in the number of signals produced by worker 1, we conclude that $n_1' \geq n_1$ and $n_2' \leq n_2$.

Second, suppose that for correlation $\rho$ there is no symmetric PEN. Then, using the *Sequential Response Algorithm*, there must be an iteration $t$ at which $n_2^t < t$ and $n_1^t = t$. Since worker 1 has taken draw $t$, it means that

$$MV(t;t-1,\rho) \geq MV(t;t,\rho).$$

Then, Lemma 6 and Lemma 7 imply that $\rho > \hat{\rho}(t,t,\gamma) > \tilde{\rho}(t,t,\gamma)$. Hence, by Lemma 5, for any correlation $\rho' > \rho$, $MV(t;t,\rho') < MV(t;t,\rho)$. Therefore, at iteration $t$ of the *Sequential Response Algorithm* when the pairwise correlation is $\rho'$, it must be that $n_2^t < t$ as well. Furthermore, it must be that $n_1^t = t$; by Lemma 4, the marginal value of draw $t$ for worker 1 is larger at correlation $\rho'$ than at correlation $\rho$. Since $(n_1^t, n_2^t)$ is asymmetric, and any asymmetric profile at any iteration of the algorithm stays asymmetric, there is no symmetric PEN.

### A.3 Proof of Proposition 2

1. We select correlations $\rho_{12}$, $\rho_{13}$, $\rho_{14}$, $\rho_{23}$, $\rho_{24}$, and $\rho_{34}$ so that there is a core allocation in which teams $(1,2)$ and $(3,4)$ form, but for which there is a matching forming teams $(1,3)$ and $(2,4)$ and a collection of PEN that strictly increases welfare. By the first two results of Proposition 1, there exists a correlation $\rho^* > -1$ below which there is a unique and symmetric PEN in which each worker produces a strictly positive number of signals and a correlation $1 > \rho^{**} \geq \rho^*$ above which there is a unique and completely asymmetric PEN in which one worker produces zero signals. We utilize these correlations in the proof.

   First, choose $\rho_{12} < \rho^*$ so that the marginal value of a signal for each worker in the unique and symmetric PEN $(n,n)$, $n > 0$, is strictly smaller than the marginal cost $c(n+1) - c(n)$. Second, choose $\rho_{34}$ such that $\rho_{12} < \rho_{34} < \rho^*$ and the unique and symmetric PEN is $(n,n)$ as well; by continuity of the marginal value of information, such a correlation is guaranteed to exist. Third, choose $\rho_{13}, \rho_{24} \in [\rho_{12}, \rho_{34}]$ close to $\rho_{12}$ so that $(n,n)$ is the unique PEN in teams $(1,3)$ and $(2,4)$. Fourth, choose $\rho_{14}$ and $\rho_{23}$ greater than $\rho^{**}$ and select an arbitrary PEN in these teams. This ensures that teams $(1,4)$ and $(2,3)$ can never form in any core matching.

   By construction, the unique PEN in teams $(1,2)$, $(1,3)$, $(2,4)$, and $(3,4)$ is $(n,n)$. Fixing this strategy profile, we observe that payoffs for any given worker are strictly decreasing in pairwise correlation,

$$\frac{\partial v(n,n;\rho)}{\partial \rho} = \frac{-2n\sigma^{-2}}{\left(2n\sigma^{-2} + (1+\rho)\sigma_\theta^{-2}\right)} < 0.$$

Hence, worker 1 and worker 2 each obtain a higher payoff together than in a team with either worker 3 or worker 4. Team $(1,2)$ must therefore form in any core matching.. This leaves worker 3 and worker 4 with no other option, but to form a team.

Notice, however, that if $\rho_{13}$ and $\rho_{24}$ are close enough to $\rho_{12}$, the gain from matching worker 1 (2) and worker 3 (4) (and selecting the unique PEN in these teams) strictly increases the total sum of utilities. Hence, we shown that the core matching is welfare dominated. Further, as incentives are strict everywhere, for $\epsilon > 0$ small, there is an $\epsilon$-ball around our chosen correlations for which the same properties are satisfied.

2. We construct an Asymmetric Effort Inefficient core allocation in which teams $(1,3)$ and $(2,4)$ form. We again select correlations $\rho_{12}$, $\rho_{13}$, $\rho_{14}$, $\rho_{23}$, $\rho_{24}$, and $\rho_{34}$. By the first two properties of Proposition 1, we can choose correlations $\rho_{14}$ and $\rho_{23}$ above $\rho^{**}$ so that teams $(1,4)$ and $(2,3)$ never form in any matching in the core. Since $\sigma^2 \geq \sigma_\theta^2$, the third and fourth properties of Proposition 1 hold. Moreover, by the third property, we can choose correlations $\rho_{13} > \rho^*$ and $\rho_{24} > \rho^*$ close to $\rho^*$ in which there is an asymmetric PEN in which each worker does strictly better as a low producer than in the unique PEN at $\rho^*$. Fix these PEN, $n^*(1,3)$ and $n^*(2,4)$, so that worker 1 and worker 4 are the low producers in each team. Let the payoff to worker 1 in this team be denoted by $u_1^*$ and the payoff to worker 4 be denoted by $u_4^*$.

   It remains to choose $\rho_{12}$ and $\rho_{34}$. We claim that for any small $\epsilon > 0$, we can find a correlation $\rho_{12} < \rho^*$ such that the payoff to each worker in the unique symmetric PEN in team $(1,2)$ is $u_{12} \in (u_1^*, u_1^* - \epsilon)$. Why does such a correlation exist? First, observe that, at $\rho = -1$, where the unique PEN is $(1,1)$, each worker obtains a higher payoff than $u_1^*$. Next, notice that at $\rho^*$, the payoff to each worker, $\underline{u}$, is less than $u_1^*$. Any payoff between $\underline{u}$ and $u_1^*$ is attainable for some $\rho \in [-1, \rho^*)$ since (i) $\rho$ decreases below $\rho^*$ payoffs increase, fixing a strategy profile and (ii) the number signals each worker produces decreases as $\rho$. (ii) implies that any discontinuities in the payoff correspondence must cause both workers to suffer a *decrease* in payoffs, as in Figure 3, thereby eliminating the possibility that the interval $(u_1^*, u_1^* - \epsilon)$ is "jumped over". A similar argument ensures we can find a correlation $\rho_{34} < \rho^*$ such that the payoff to each worker in the unique symmetric PEN in team $(3,4)$ is $u_{34} \in (u_4^*, u_1^* - \epsilon)$.

We argue that the matching $\mu$ satisfying $\mu(1) = 3$ and $\mu(2) = 4$, together with the equilibria $n^*(1,3)$ and $n^*(2,4)$ is a core allocation. By construction, worker 1 (worker 4) is strictly better off playing $n^*(1,3)$ ($n^*(2,4)$) than if they formed a team with worker 2 (worker 3) and played the unique PEN in this team.

However, for $\epsilon > 0$ small enough, the total sum of utilities is higher when matching worker 1 with worker 2 (worker 3 with worker 4) and playing the unique PEN in that team. Hence, the constructed core allocation is welfare inefficient. Moreover, in the welfare-improving teams, worker 1 and worker 4 each exert relatively more effort than their partners in the original core allocation. Hence, the core allocation we constructed is Asymmetric Effort Inefficient. Again, as incentives are strict everywhere, for $\epsilon > 0$ small, there is an $\epsilon$-ball around our chosen correlations for which the same properties are satisfied.